Chapter 7

Refining Coordination Problems

7.1 The return of coordination problems

Having already touched in Chapter 6 upon the role played by the 'recurrent situation S' of coordination in Lewis' definition of convention we may first stop to consider whether Lewis' motive for substituting this for the characterization in terms of coordination problems was right. This question is important since, while the formulation in terms of a recurrent situation of coordination encompasses all the sample conventions, it also gives up all the intuitive, technical and theoretical understanding that had been developed on the basis of the Lewis' game theoretic approach. The first question to ask then is whether Lewis was right in kicking the ladder out from under himself in this way?

Recall, Lewis' reason for writing out coordination problems of the definition was that supposedly some conventions could not be reasonably represented as coordination problems. In particular, he used the example of the contended oligopolists who set prices without explicit agreement. But what specifically was the kind of convention followed by the oligopolists that could not be represented? Lewis gives us one example that might apply; they could "follow a price leader: one firm that takes the initiative in changing prices, with due care to set a price in the range that is satisfactorily to all of [them]." However, according to Lewis the problem posed by such conventions were that prices could be changed at any time and hence the problem could not be represented as a 'discrete' and self-contained coordination problem. Given the situation faced by the oligopolist as it is described by Lewis (1969:46-47), he is obviously right that it cannot be represented properly as a self-contained coordination problem. Yet, following (Syverson 2003:32-34) it is possible to re-describe the contended oligopolists' situation in a way that readily allows for such representation if '*getting the strategies right*'.

Getting the strategies right In *Convention* a coordination problem was defined as a situation "of interdependent decision by two or more agents in which coincidence of interest predominates and in which there are two or more proper coordination equilibria" (Lewis 1969:24). In contrast, the oligopolists' problem was that of maintaining a uniform but fluctuating price without making any explicit agreement. In order for there to be a convention in this problem, there must be alternative strategies for the firms to solve this problem; in particular, there must be strategies that combine so as to instantiate proper coordination equilibria. One of these alternatives is in fact offered by Lewis himself: (1) follow a price leader i, where i takes the initiative in changing prices with due care to set a price in the range that is satisfactory to all of them. If we then assume that there are three firms that might serve as price leaders this yields three alternatives. But there are also further alternatives. For instance: (4) Follow a price leader i, where i is whichever firm currently happens to take the initiative in setting a new price, with the default rule that if more than one firm seems to take the initiative, each firm should follow the one setting the highest price. In fact, an abundance of even further alternatives may easily be given – in particular, any alternative attaching choice to some public signal – but here the above will suffice.

Now, what is important to notice is that there is nothing to prevent us from assuming that each of the four alternatives satisfy the criteria for being proper coordination equilibria such that each firm finds one of these alternatives at least as acceptable as any other in which all but one follow the convention, and each one strictly prefers this to anything it could achieve assuming that all the other firms conform. Finally, to assure fluctuation it may also be assumed that the price leader can change his price once in a while, even when such a price leader were to lower the price.

Returning to the notion of a self-contained problem, Lewis points out in *Convention* that if a situation "is a self-contained problem of interdependent decision, in which each agent involved makes one choice of action and the outcome of each depends on the actions of all… *then* [this situation] is a coordination problem and uniform conformity to R is a coordination equilibrium" (Lewis 1969: 68-69). But given this definition, plus (1) – (4) above it becomes obvious that the example of the contended oligopolists can be represented as a self-contained coordination problem as in figure 262.2 below, where $u(c) > u(f)$, agent 1 chooses between rows, agent 2 chooses between columns and agent 3 chooses between matrixes. In fact, given some further details, such a representation may be argued to be the possible for any situation satisfying the 'recurrent situation' formulation, cf. (Syverson 2003:35).

Figure xx.x: *Contented oligopolists:*

Oligopolist 2 (columns); *Oligopolist 1* (rows)

Ol. 3, *1*	*1*	*2*	*3*	*4*
1	c	f	f	f
2	f	f	f	f
3	f	f	f	f
4	f	f	f	f

Oligopolist 2 (columns); *Oligopolist 1* (rows)

Ol. 3, *2*	*1*	*2*	*3*	*4*
1	f	f	f	f
2	f	c	f	f
3	f	f	f	f
4	f	f	f	f

Oligopolist 2 (columns); *Oligopolist 1* (rows)

Ol. 3, *3*	*1*	*2*	*3*	*4*
1	f	f	f	f
2	f	f	f	f
3	f	f	c	f
4	f	f	f	f

Oligopolist 2 (columns); *Oligopolist 1* (rows)

Ol. 3, *4*	*1*	*2*	*3*	*4*
1	f	f	f	f
2	f	f	f	f
3	f	f	f	f
4	f	f	f	c

Consequently, we can conclude that Lewis seems to been wrong in discarding the notion of *coordination problems* as the central notion is his theory. By 'getting the strategies right' for the kind of recurrent situations satisfying conditions (3.)-(5.) in Lewis' definition of convention, any 'recurrent situation of coordination' may be reconstructed as a coordination problem in the sense originally targeted by Lewis. But not only does this does this re-install the notion of a coordination problem at the root of Lewis' theory of convention as the proper representation of the strategic aspects of those situations the prevalence of which throughout everyday social interaction are thought to give rise to social conventions. It also allows for retaining all of the intuitive, technical and theoretical understanding that was originally developed by Lewis within the game theoretic framework. That is, recurrent coordination games may be used to represent the very *raison d'être* of conventions, so to say. Thus, the original definition of a coordination problem may be kept as the basic one of interest:

> **Definition 95.1** *Coordination problem*: a situation of interdependent choice by two or more agents in which co-incidence of interest predominates and in which there are two or more strict coordination equilibria.

Coincidence of interest Now, recall that Lewis arrives at this notion of a coordination problem and the requirement that a convention has to instantiate a strict coordination equilibrium in such a problem by way of analyzing his sample coordination problems and the conventions they give rise to. Naturally he takes it that the samples are representative for the general class of conventions and their distinctive features important for an understanding of what characterizes this class (Lewis 1969:24). A central question to ask then is whether his samples are in fact representative and whether the notion of a coordination problem actually captures the relevant features?

Begining the examination with the feature of 'coincidece of interest', Lewis takes as a necessary feature of a coordination problem that such coincidence of interest predominates in the coordination problem. More precisely, this is taken to be satisfied by games in which the differences between the different agents' payoffs for any one action combination are small compared to some of the differences between payoffs for different action combinations (Lewis 1969:14). However, there are two problems with this. Originally, as was mentioned in Chapter 5, Lewis does not take the feature of predominant coincidence of interest as a general one of coordination problems. He merely chose to confine his attention to situations with this feature for what seems to be pragmatic considerations, cf. (*ibid*). Hence, asserting it as an integral part of the most important concept in his theory seems unwarranted. Second, by invoking "the differences between the different agents' payoffs for any one action combination" it presupposes the kind of interpersonal comparison of payoffs irreconcilable with any standard interpretation of game theory.

That aside, by the end of his discussion of coordination problems in *Convention* Lewis chooses to reformulate this requirement in terms of an *agreement-in-preferences* clause, which he takes to capture the same feature (Lewis 1969:69). Accordingly, it is now to be required that every agent must have approximately the same preferences regarding all possible combinations of actions. But ultimately this does not ring true since it would disqualify, or at least make the following game less of a coordination problem:

Matrix 264.1:

		agent 2	
		α	β
agent 1	α	$\mathbf{9, 1}$	$0, -10$
	β	$-10, 0$	$\mathbf{1, 9}$

Though one may have quite firm expectations of which convention would emerge in this game, it obviously still presents a coordination problem that needs to be solved. But insisting on its status as a coordination problem, as one surely must, implies that the agreement-in-preferences clause is too strong when defining the relevant features of coordination problems.

Whatever the reason, it is thus fortunate that neither requirement appears in the revised 1975-definition. There, Lewis seems to think instead that a mere presence of multiple strict coordination equilibria is sufficient to distinguish convention from deadlocked conflict (Lewis 1975:16). In particular, he takes it that the coordinative nature of these equilibria, as expressed in condition (4.) of the 1975-definition, suffices to characterize the kind of coincidence of interest in question. This since, contrary to the kind of situations of conflict represented by constant-sum games, this property of equilibria ensures that no one else stands to benefit if anyone comes to deviate.

In conclusion then, it seems that any viable concept of a coordination problem should not rely on any considerations based on the predominance of coincidental interest or some kind of agreement-in-preferences. Instead it should be based solely on the idea of a presence of multiple strict coordination equilibria. Thus the relevant definition of a coordination problem may be revised to the following:

Definition 96.1 *Coordination problem*: a situation of interdependent choice by two or more agents in which there are two or more strict coordination equilibria.

Strict coordination equilibria, an ambiguity Obviously, this revision puts the notion of strict coordination equilibrium at the center of attention. Now recall, that Lewis defines a *strict coordination equilibrium* as a strict Nash equilibrium from which no one stands to benefit if anyone comes to deviate. While not stating this explicitly, it seems clear that Lewis' motive for taking this notion as one of the essential features of a coordination problem derives from Schelling's preliminary exposition of it in *The Strategy of Conflict* (1960). In particular, it seems to be intended for capturing Schelling's idea that such problems call for a 'meeting of the minds' and a communication of intentions, rather than the kind of secrecy or concealment at play in situations of conflict, cf. (Schelling 1960:83,86,106) and (Sugden & Zamarrón 2006).

While if taken in isolation the notion of a strict coordination equilibrium is clear and unambiguous, looking at Lewis' use of it and comparing this with his later 1975-definition an ambiguity arises concerning the notion of coordination involved, see e.g. (Gilbert 1981).

Literally taken Lewis' notion of coordination equilibrium does not require that anyone becomes *worse off* if anyone deviates. What matters is that nobody would be *better off* had any one agent acted otherwise. This *weak* notion of coordination equilibrium is the one assumed by Lewis to begin with in *Convention*. However, as *Convention* progresses and as it finally becomes evident in the 1975-definition, he ends up assuming a *strong* notion of coordination equilibrium. This requires that someone do in fact become worse off if anybody deviates from such a profile. In particular, it should be noted that it is this strong notion that allows Lewis to point out a feature making conventions norms by definition: any deviation from such an equilibrium implies making oneself as well as others worse off, see (Lewis 1969:97-100). But while this is implied in the later part of *Convention* by the requirement that every agent has approximately the same preferences regarding all possible combinations of action, in *Languages and Language* where the agreement-in-preferences clause is left out, the strong version is implied by condition (4.) stating that "there is a general preference for general conformity to R rather than slightly less-than-general conformity — in particular, rather than conformity by all but any one" (Lewis 1975:5).

This could seem to suggest that the notion to be opted for as the feature relevant for characterizing the coordination problems giving rise to conventions should be the *strong* one. However, at least one of Lewis' sample coordination problems may be argued to fall short of the intended characterization if the strong notion of coordination is to be the relevant one. This is the stag hunt case inspired by *Rousseau's Discours sur l'inégalité* and paraphrased by Lewis as follows:

> "Suppose we are in a wilderness without food. Separately we can catch rabbits and eat badly. Together we can catch stags and eat well. But if even one of us deserts the stag hunt to catch a rabbit, the stag will get away; so the other stag hunters will not eat unless they desert too. Each must choose whether to stay with the stag hunt or desert according to his expectations about the others, staying if and only if no one else will desert" (Lewis 1969:7).

This scenario makes for assuming that any agent i in the wilderness have the following preferences over possible outcomes arising from combinations of s, *hunting stag*, and r, *hunting rabbit*: $(s_i, s_j) >_i (r_i, s_j) \sim_i (r_i, r_j) >_i (s_i, r_j)$. Given suitable ascriptions of utilities this scenario may then be represented by the *stag hunt game* below:

Matrix 267.1: *stag hunt game*

		agent 2	
		s	r
agent 1	s	**3, 3**	1, 2
	r	2, 1	**2, 2**

Now, if the concept of a coordination equilibrium is to be interpreted in the strong sense, the example of Rousseau's *stag hunt* does not qualify as a coordination problem since it only features a single coordination equilibrium in the strong sense — (s_i, s_j). This then points to a weak interpretation of coordination equilibrium; and though its appearance early on in *Convention* may seem to undercut its relevancy, Margaret Gilbert (1981:47) reports that

Lewis held the weak notion as the relevant one as late as in 1977. But whether Lewis ultimately embraced the weak or strong interpretation of coordination equilibrium it turns out that a more relevant preemptive question to ask is whether one should accept the notion of coordination equilibrium as a relevant one, in whatever sense.

7.2 Rejecting coordination equilibria

As mentioned, Lewis seems to incorporate the idea that the equilibria involved in coordination problems need to be coordinative in order to capture Schelling's observation that in such problems no one has an interest in deception, secrecy or concealment. But whether or not one chooses the weak or the strong formulation of coordination equilibrium as the relevant one for defining coordination problems, it turns out that such a requirement ultimately rules out as conventional certain paradigmatic cases. In particular, it rules out exactly the kind of property rights that Hume originally devised his theory of convention to account for (Section 2.3); the very theory the spirit of which Lewis claims his theory is true to.

To see this consider what would be a standard recurrent situation giving rise to property rights taken from (Walmsley 1932), recounted by and quoted from (Sugden 1989:85) and equivalent to one of Lewis' sample coordination problems giving rise to a convention, see (Lewis 1969:7,47-48):

> In a fishing village on the Yorkshire coast there used to be an unwritten rule about the gathering of driftwood after a storm. Whoever was first onto a stretch of the shore after high tide was allowed to take whatever he wished, without interference from later arrivals, and to gather it into piles above the high-tide line. Provided he placed two stones on the top of each pile, the wood was regarded as his property, for him to carry away when he chose. If, however, a pile had not been removed after two more high tides, this ownership right lapsed.

Obviously, a rule like this qualifies as a convention of property rights on the Humean theory of convention as well as in the eyes of Lewis.

The underpinning situation giving rise to it this convention may, however, be given a game theoretical interpretation familiar from the previous chapter. Each agent has a choice between two pure actions: claim, i.e. to claim the driftwood, and *refrain*, i.e. to refrain from claiming the driftwood. This gives rise to four possible action-combinations for an encounter between two agents: (*claim*, *claim*) where both try to claim the driftwood resulting in a costly or risky dispute or conflict; (*refrain*, *refrain*) where both refrain from claiming the driftwood as theirs and finds instead a suitable division of the driftwood; (*claim*, *refrain*) and (*refrain*, *claim*) where one claims the driftwood while the other one refrains. Also, it is quite reasonable to assume that each agent i gathering driftwood on the beach has the following preference ordering over these states: $(claim_i, refrain_j) >_i (refrain_i, refrain_j) >_i (refrain_i, claim_j) >_i (claim_i, claim_j)$. Given suitable ascriptions of utilities this scenario may then be represented by the familiar *Hawk-dove game* below:

Matrix 268.1: *Hawk-dove*

agent 2

agent 1		refrain	claim
	refrain	2, 2	**1, 3**
	claim	**3, 1**	0, 0

But notice now that this game does not contain even a single coordination equilibrium in either the weak or the strong sense. Thus, while the game seems a reasonable representation of the situation underpinning the kind of property rights targeted by Hume's theory of convention, it does not qualify as a Lewisian coordination problem.

Of course a possible way to deal with this problem is by adjusting intuitions so that practices underpinned by preference structures falling in the category of games like *hawk-dove* game do not qualify as conventions. Doing this seems somewhat arbitrary though, since it will not only rule out the Humean ambition in the theory of convention, but also several other social practices otherwise thought of as conventional. For instance, practices for the distribution of goods such as those based on *time based procedures* (queuing up for cinema tickets, the bus, or a kidney, the last-in-first-out principle used when laying off people, or using original appropriation to determine ownership as when people leaves some of their belongings at their table or train seat to claim temporary use-right when going to the toilet) as well as practices based on *lottery based procedures* (playing stone, paper, scissor in order to determine who should take out the trash, drawing in a lottery in order to determine who gets access to a new medical drug first, or drawing straws as to who's company should land on the beaches first in an attack) as well as practices based on *need based procedures* (access to expensive medical drugs for permanent illnesses, systematic distribution of relief aid, or access to additional teaching in public schools). Thus, it may seem more reasonable to stick with the original intuition evaluating Humean property rights as true cases of convention.

But apart from intuitions, the same conclusion may also be argued from more reliable premises – those based on the theory of rational agency in interaction, i.e. the theory of games. This argument proceeds as follows.

As Chapter 4 showed game theory focuses on explaining the behavior and choices of decision makers in situations of strategic uncertainty. What is taken to determine behavior and choices within this framework is the preferences held by agents over possible outcomes. In particular, such preferences are taken to include *all* relevant aspects of the outcome that the agent cares about. Together with the agents' expectations these preferences in turn determines the agents' instrumental preferences over actions, i.e. the kind of preferences determining for each agent which action to be taken; we may call such preferences *action-determining preferences*.

However, Lewis' notion of coordination equilibrium incorporates another kind of preferences *viz.* what may be labelled *non-action-determining preferences*. Following (Miller 1986:125) "A preference is non-action-determining if it makes no differences to anyone's actual actions, or to what anyone's actions would have been if others had acted differently." It should be noted though that this notion of Miller's is ill-defined. Since the notion of preference is unavoidably tied to action, or a disposition to action, the notion of a *non-action determining preference* makes strictly speaking no sense as a preference *per se*. Thus, non-action determining preferences are always to be understood relative to the particular preference structure embedded in the situation in question.

Thus since non-action-determining preferences do not determine or influence actions in the given situation, they are not relevant for any explanation of behavior. This in turn implies that whether a practice instantiate a coordination equilibrium or just an equilibrium makes no difference in the kind of explanation involved in explaining the behavior, actions or choices in question. It is for these more reliable reasons that Lewis' requirement that the equilibria instantiating conventions must be coordinative may be argued to be arbitrary as well as irrelevant.

As a consequence it only seems reasonable to reject Lewis' requirement that the equilibria in question has to be coordinative — whether taken in the strong or the weak sense. Whether a practice instantiates such an equilibrium or not does not seem relevant for its conventionality in so far as conventions are taken to be explained in terms of their coordinative role in settling strategic uncertainty on the basis of precedent.

Still, there is a distinction worth making in between those conventions which are underpinned by coordination equilibria and those which are not. For one, the former kind captures Schelling's idea of the absence of reasons for deceit that may turn out to be important in understanding certain problems and discussions surrounding such conventions. Second, the distinction may be used as a component hypothesis in a broader theory about the nature and dynamics of social norms, see e.g. (Sugden 2000). It is for such reasons that it may be worth distinguishing between what may be considered as *trivial conventions*, i.e. conventions underpinned by coordination equilibria (without this has anything to do with Lewis' considerations concerning 'trivial games'), and *non-trivial conventions*, i.e. conventions not underpinned by *coordination equilibria*. Yet, the important thing to notice is that the notion of coordination equilibrium should be abandoned as part of what defines a coordination problem. This leaves us with the following definition:

> **Definition 271.1** *Coordination problem*: a situation of interdependent choice by two or more agents in which there are two or more strict equilibria.

7.3 The problem of strict equilibria

Given the abandonment of the idea that the equilibria underpinning conventions need to be coordinative one may come to wonder whether Lewis' other requirement concerning the relevant equilibria is unreasonable as well. That is, one may question whether and under what conditions coordination problems necessarily involve strict equilibria.

The first thing to be noted is that it is evident throughout *Convention* that Lewis not only assumes that conventions are to be modeled as equilibrium configurations, but also that these necessarily are strict (or 'proper' as he calls it), see especially (Lewis 1969:16). His main reason for this requirement seems to be that the presence of multiple strict equilibria is an essential requirement for the kind of real world coordination problems to exist which he takes to provide the *raison d'être* of social convention; a requirement that according to Lewis will not be satisfied if merely requiring the presence of a multiplicity of equilibria. Lewis' argument for this latter point may be illustrated using the game below.

Matrix 271.1:

agent 2

agent 1		a	b
	a	**1, 1**	**1, 1**
	b	0, 0	0, 0

According to Lewis the problem with this game is that it exhibits a kind of triviality where equilibrium 'will be reached if the nature of the situation is clear enough so that everybody makes the best choice given his expectations, everybody expects everybody else to make the best choice given his expectations, and so on' (Lewis 1969: 22). A kind of triviality that Lewis argues is akin to the kind of triviality exhibited by any game that may be solved by the iterated deletion of dominated strategies, cf. (*ibid*: 16-17). Consequently, in such situations there is no need for the coordinative function of social conventions. That is, in such situations a convention would be superfluous since it would play no role that common knowledge of rationality could not handle by itself. This, then, cannot be the kind of situation explaining the existence of convention. Instead, that kind of situation must by necessity be of a kind where strategic uncertainty remains even under the assumption of common knowledge of rationality. Hence games like the above should be excluded.

However, contrary to what Margaret Gilbert claims in 'Game Theory and Convention' (Gilbert 1981:43), Lewis recognizes well that this does not amount to the same thing as saying that any game will be strategically trivial in the absence of multiple strict equilibria. For instance he considers the following game:

Matrix 272.1:

agent 2

agent 1		α	β	γ	δ
	α	$\mathbf{1,1}$	$\mathbf{1,1}$	$0,0$	$0,0$
	β	$\mathbf{1,1}$	$\mathbf{1,1}$	$0,0$	$0,0$
	γ	$0,0$	$0,0$	$\mathbf{1,1}$	$\mathbf{1,1}$
	δ	$0,0$	$0,0$	$\mathbf{1,1}$	$\mathbf{1,1}$

In a game like this none of the equilibria are strict. Yet, the decision by each agent clearly depends on what he expects the other agent to do. In particular, it is not clear what to expect in this game even if it is known that everyone are expected to make their best choice given their expectations, that everybody expects everybody else to make their best choice given their expectations, and so on. Hence, either strict equilibria are not a necessary condition for a coordination problem to obtain, or the requirement that the coordination problems underpinning social conventions should exhibit strict equilibria should be understood as a matter of interpretation, rather than as a trivial fact given the nature of the strategic uncertainty in question.

Thus, though this is usually overlooked, Lewis implicitly assumes that a more fundamental feature characterizes the coordination problems giving rise to social conventions and which the requirement of strict equilibria is aimed to capture *viz.* that the agents not only base their choices upon their expectations about the choices of others, but also *calls for them to try for the same equilibrium* (Lewis 1969:21). Otherwise, he claims, there would be 'no need for coordination' (*ibid.*). Ultimately this appears to be a crucial premise in his reason for choosing to "stipulate… that a coordination problem must contain at least two proper [i.e. strict] coordination equilibria" (*ibid.*:22). Though it seems to be only dimly recognized by Lewis himself, it follows that when an equilibrium is strict agents will by definition be taking each other to aim for the same equilibrium profile in so far as that equilibrium is their best choice given what they expect the others to do and that rationality is common knowledge. That is, if they hold expectations making the pursuit of that equilibrium the best choice, these expectations are also expectations that take the other(s) to aim for that very same equilibrium. Thus, Lewis does not merely assume that coordination problems are characterized by strategic uncertainty — i.e. situations in which agents base their choices upon their expectations about the choices of others when rationality is common knowledge. Rather he assumes that coordination problems are *situations where agents base their choices upon expectations about the choices of others and in doing this they aim, and will take each other to aim, for the same equilibrium* — an assumption that also may be noticed to fit well with the characteristic of convention stated in Chapter 2 saying that agents act to achieve the same purpose. This, of course, still leaves the problem of what to say about games like that above; should it not be considered as a coordination problem merely by stipulation?

Lewis' answer to this question turns out to be a pragmatic one. To avoid cases like that presented by the game above Lewis (1969:22-23) makes use of a standard simplifying assumption in game theory based on the reduction of payoff equivalent strategies, cf. Chapter 4. In the above game this means that strategies α and β in S_i are collapsed into a single strategy a and strategies γ and δ in S_i are collapsed into a single strategy b. Doing this then reduces the game to the by now well-known *driving game*:

Matrix 274.1: *the driving game*

agent 2

agent 1		a	b
	a	**1, 1**	0, 0
	b	0, 0	**1, 1**

As mentioned this solution may be taken as a somewhat pragmatic or even as an *ad hoc* one. It is true that given the reduction of payoff equivalent strategies the game now features two strict equilibria. Yet, this simplifying assumption may seem to be valid for purposes of strategic analysis only — i.e. for the purpose of recommending strategies, rather than explaining behavior. While the two games may be strategically equivalent, it does not follow that choosing between strategies α, β, γ, and δ on the one hand and between a and b on the other are also equivalent from the perspective of the agents since the former distinguish between the actions that the latter has collapsed.

Musical chairs Lewis seems somewhat to recognize this. In particular, he admits that the resulting stipulation requiring coordination problems to exhibit at least two strict equilibria is only the strongest out of several possible restrictions, but a "satisfactory restriction would be complicated and would entail too many qualifications later" (*ibid.*:22-23). Still, he argues, that situations without any strict equilibria like that below which "are ruled out by the strong restriction, and ought to be ruled out, cannot be rescued by any such consolidation" (*ibid.*:22).

Matrix 275.1: *musical chairs*

agent 2

agent 1

	α	β	γ
α	$0,0$	$\mathbf{1,1}$	$\mathbf{1,1}$
β	$\mathbf{1,1}$	$0,0$	$\mathbf{1,1}$
γ	$\mathbf{1,1}$	$\mathbf{1,1}$	$0,0$

But 'ought' such situations really be ruled out? If not, Lewis' requirement of strict equilibria could seem too strong.

Think of *musical chairs* as follows. In this game agent 1 and 2 both want to coordinate so that they do not end up choosing one and the same chair, but are both indifferent about which chair they end up sitting upon. Obviously, in this game none of the equilibria are strict. Nor is any of the strategies payoff equivalent. This also implies that the agents do not prefer to coordinate on the same equilibrium as Lewis would require of a coordination problem. Yet, the game still appears to present a clear-cut coordination problem. In particular, it would seem that conformity to a regularity where agent 1 always chooses α and agent 2 always chooses either β or γ would serve to coordinate the agents' actions and that this could readily be conceived of as a case of a convention.

Before drawing any conclusion from this, though, it should be considered what Lewis might have said in defense of his stipulation of multiple strict equilibria in light of this example. For instance, it may be thought that he would have argued along the usual contemporary practice in game theoretical modeling that one has failed to specify the strategies of the game correctly. In particular, the regularity may be described as one instantiating one of multiple available strict equilibria in a coordination problem if one specifies the available strategies along the lines of what seems to be the agents choice-situation in which $S_1 = \{\alpha, \beta, \gamma\}$ and $S_2 = \{(\alpha \vee \beta), (\alpha \vee \gamma), (\beta \vee \gamma)\}$, where each of agent 2's disjunctive strategies put an equal weight on each component strategy. This re-description yields the *revised musical chairs* below.

Matrix 276.1: *revised musical chairs*

agent 2

agent 1

	$\alpha \vee \beta$	$\alpha \vee \gamma$	$\beta \vee \gamma$
α	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$	$\mathbf{1}, \mathbf{1}$
β	$\frac{1}{2}, \frac{1}{2}$	$\mathbf{1}, \mathbf{1}$	$\frac{1}{2}, \frac{1}{2}$
γ	$\mathbf{1}, \mathbf{1}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$

Given such a re-description, then, it could appear as if Lewis' stipulation suffices.

However, appearances to the contrary, closer scrutiny reveals that any argument relying on such a re-description is ultimately *ad hoc*. Though it retains observed behavior as equilibrium behavior, the re-description in question is neither one based on the mere re-labelling of strategies, nor one based on the collapsing of payoff equivalent strategies. Rather it introduces whole new strategies into the game; or more precisely, it gives an alternative description of the recurrent choice-situation faced by the agents. To see the *ad hoc* nature of this re-description one merely has to notice that no reason has been provided why strategies like $(\alpha \vee \beta)$, $(\alpha \vee \gamma)$, and $(\beta \vee \gamma)$ should be excluded from S_1; and, likewise, why strategies α, β, and γ, should be excluded from S_2. But if allowing for these strategies, as shown in the matrix below, it now turns out that once again there are no strict equilibria in the game.

Matrix 276.2: *musical chairs 3*

agent 2

		α	β	γ	$\alpha \vee \beta$	$\alpha \vee \gamma$	$\beta \vee \gamma$
	α	$0, 0$	$\mathbf{1, 1}$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$	$\mathbf{1, 1}$
	β	$\mathbf{1, 1}$	$0, 0$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$
agent 1	γ	$\mathbf{1, 1}$	$\mathbf{1, 1}$	$0, 0$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$
	$\alpha \vee \beta$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{4}, \frac{3}{4}$	$\frac{3}{4}, \frac{3}{4}$
	$\alpha \vee \gamma$	$\frac{1}{2}, \frac{1}{2}$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{4}, \frac{3}{4}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{4}, \frac{3}{4}$
	$\beta \vee \gamma$	$\mathbf{1, 1}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{4}, \frac{3}{4}$	$\frac{3}{4}, \frac{3}{4}$	$\frac{1}{2}, \frac{1}{2}$

This problem turns out to run even deeper than one might have thought at first. It reveals that whether or not a a situation is taken to exhibit, as well as whether or not a particular regularity instantiates, a strict equilibrium may ultimately depend upon the way we *choose* to describe a given decision problem. Only if one particular description is favored in this choice can one insist that coordination problems should necessarily exhibit (multiple) strict equilibria. Yet, Lewis has already rejected that any description is more natural than any other in so far as it preserves the relevant equilibria of the game; that is, the equilibria which are taken to be instantiated by the possible regularities considered by the game theorist.

Lewis' re-description of telephone tag In fact, this point is nicely illustrated by Lewis' 're-description' of the *telephone tag game* (Lewis 1969:11). By this re-description Lewis is not merely assigning new labels to the strategies of the original *telephone tag game*, nor just collapsing payoff equivalent strategies. Rather, as is evident from what was said about heterogenous conventions in the previous chapter, he is actually re-conceptualizing the game in terms of whole new strategies; the unconditional strategies 'call' and 'wait' are clearly very different from the strategies of α : 'call if you're the original caller, wait otherwise' and its inverse counterpart β : 'wait if you're the original caller, otherwise call' which both conditionalize the original actions on an arbitrary feature of the world that did not figure as part of the original game. However, Since Lewis' claim is that there is no *natural description* of strategies and since many different arbitrary features are bound to exist in *any instance* where are phone call is cut off, there is nothing to prevent one to introduce strategies conditionalizing on any other arbitrary feature presenting itself in such an instance, such as *the age of the participants* yielding conditional strategies such as γ : 'call back if you're the oldest one, otherwise wait' and the inverse counterpart, δ, of this strategy. In fact, the number of available equilibria grows exponentially with the number of arbitrary features available in any given situation. This ultimately implies that for *any individual instance* where two agents wants to reestablish a cut of phone call, no strict equilibria exist.

Matrix 277.1: *Telephone tag*

		agent 2					
		call	*wait*	α	β	γ	δ
	call	0, 0	**1, 1**	**1, 1**	0, 0	**1, 1**	0, 0
	wait	**1, 1**	0, 0	0, 0	**1, 1**	0, 0	**1, 1**
agent 1	α	0, 0	**1, 1**	**1, 1**	0, 0	**1, 1**	0, 0
	β	**1, 1**	0, 0	0, 0	**1, 1**	0, 0	**1, 1**
	γ	0, 0	**1, 1**	**1, 1**	0, 0	**1, 1**	0, 0
	δ	**1, 1**	0, 0	0, 0	**1, 1**	0, 0	**1, 1**

Ultimately, this problem turns out to be fully generalizable to all situations presenting a coordination problem. For instance, even for any instance of the driving game it turns out to feature no strict equilibria since nothing justifies the exclusion of such strategies as 'keep to the side of the weel' or 'keep to the side nearest your hometown' as well as their counterparts. Further, there is nothing to prevent that agents may be ascribed strategy sets which includes strategies that conditionalize on any available feature in the situaiton. Again considering the driving game this means that there is nothing to exclude strategies such as 'keep left in daytime, but right at night' or even 'keep left if you're happy, but right if you're not', plus counterparts.

Salience underdetermined In fact, given some consideration this problem becomes even worse. To see why, notice that *time* itself may deliver the states of affairs for conditionalization on, thereby giving rise to the possibility of introducing strategies such as 'keep left on workdays, and right in weekends', or even '*keep left* until time n, then *keep right* in all future'. Especially the latter type of strategy raises a devastating problem for the theory of convention as portrayed so far. If strategies of the form 'do s_i' until time n, then do s_i''' are allowed, then there will not only always be a possible strategy such that no strict equilibrium exists; it will also be impossible for any inductive agent ever to decide what strategy other agents are following as well as have been following in the past, and hence also which strategy-profile is made salient by succesfull precedent.

As an example of this problem, consider the situation $n+1$ where agent i has succesfully coordinated by following the strategy $s_i' = (\textit{keep left})$ a 1.000 times when encountering agent j in the recurrent problem of coordination arising from approaching each other in the traffic. Now, while it seems natural to take this as evidence for i that j is following s_i' as well, nothing has been said so far that allows i to rule out the possibility that agent j has actually been following the strategy s_i'' = ('*keep left* until time n, then *keep right* in all future'). But if this is the case, then any series of observations of past succesfull coordination will provide equal evidence for s_i' and s_i'', respectively. Consequently, any successful precedent in the sense of a series of successful combinations of actions may be projected in infinitely many ways into the future and any combination of actions in an ensuing stage game will be salient under some description of the precedented series of combinations.

Still, for Lewis' purposes his choice of redescription is perfectly acceptable. Recall, Lewis objective is to analyse the concept of convention, not to explain how conventions come about or why agents actually conform. This means that he is allowed to choose any description that succeeds in giving an adequate description and possibly also prediction of a behavioral regularity in conformity with an existing convention. Lewis' re-description of *telephone tag* does exactly that; and so does *musical chairs* as represented in matrix 276.1. Further, his assumption of salience by 'natural analogy' succesfully by-passes the challenge posed by salience being underdetermined. As Lewis recognize, "Were it not that we happen uniformly to notice some analogies and ignore others – those we call 'natural'and 'artificial,' respectively — precedents would always be completely ambiguous and worthless"(ibid:37-38). Thus, while this approach may turn out to be problematic when seeking to explain the emergence of, and conformity to, social conventions (cf. Chapter 12), Lewis' requirement that coordination problems exhibits multiple strict equilibria suffices for his purposes and consequently, definition 271.1 may be retained.

7.4 The insufficiency of multiple strict equilibria

But is the multiplicity of strict equilibria also *sufficient* for characterizing a coordination problem? While voices are parted, the contemporary line in the mathematically inclined game theoretical literature on convention seems to think so without much consideration going into the question, see e.g. (Binmore 2006) and (Rescorla 2007). Yet, dissenting points of views have been raised by more philosophically inclined theorists, see e.g. (Miller 1986), (Syverson 2003) and **(Ross 2006)**. Here an argument is given in support of the latter positions.

To see why the multiplicity of strict equilibria is not a sufficient condition for identifying a coordination problem consider again the stag hunt game previously encountered. In this game there are two strict equilibria: $(s,\ s)$, $(r,\ r)$.

Matrix 280.1: *the stag hunt*

agent 2

agent 1		s	r
	s	**3, 3**	1, 2
	r	2, 1	**2, 2**

Still, it is not a coordination problem in the usual sense of trying to coordinate on contingent means in order to achieve one and the same objective O.[1] Indeed, in the equilibrium (s, s) they are trying to coordinate with each other on hunting the stag. However, in the other equilibrium (r, r) they are not trying to do so, but are rather 'going their separate ways'. That is, they

[1] (Miller 1986) refers to O as a 'collective end'. Here this terminology is rejected since it may seem to imply that the involved agents necessarily are aware that they share this end which ultimately does not square with earlier discussions of such conventions as the one of *smoking in the schoolyard*.

do not care what the other one is doing and hence talking about 'coordination' in the relevant sense would seem to be misleading. Further, this is obviously not a consequence of the equilibria being strict in the case at hand. A similar case could easily be constructed in a game where no strict equilibria exist. In conclusion, then, the requirement of multiple strict equilibria does not ensure the existence of a coordination problem in the proper sense.

A straightforward remedy for this problem is to require that in a coordination problem multiple strict equilibria must be present for achieving one and the same collective end or shared objective O. In particular this is exactly what the pre-theoretical characterization of a convention as well as Lewis' comment on (Lewis 1969: 21-22) also seem to require us to do. Yet, others like Seamus Miller has argued that such a move actually reveals that it is the idea of a shared or collective objective which is the central feature.

Miller on collective ends The conclusion that (Miller 1986) draws from introducing this additional requirement is that conventions may exist that do not solve coordination problems. That is, conventions are not connected to coordination problems by necessity. The first step in his argument distinguishes between what he calls *formal* and *informal coordination problems*. While the former are problems merely featuring multiple equilibria, such as in the stag hunt game, the latter are problems of coordinating on one of multiple available equilibria comprisinig strategies for achieving one and the same objective. Next, he retains that behavior instantiating equilibria such as the (s, s) equilibrium in the Stag hunt game should be regarded as a convention despite no informal, but only a formal coordination problem is present. His reason for this may seem quite reasonable:

> " ...the conditional preferences and the interdependence of action is in fact the same for both regularities which solve informal coordination problems and for certain other regularities which don't solve this type of problem. This is presumably a motivation for viewing both types of regularities as belonging to the same basic category." (Miller 1986: 137)

Miller then mentions several problems falling within this category: the problem of whether to join an oligopoly or just compete for prices, whether to share the spoils after a robbery or fighting over it, and what he calls the 'convention not to mislead people' which has only a non-conventional, but no conventional alternative, *viz.* misleading people. Miller then claims that all of these problems may provide the *raison d'être* of practices that all fall within the class of phenomena referred to as 'conventions' in everyday language and cite Lewis' own use of this concept in support.

This, however, flatly contradicts the pre-theoretical characterization of Chapter 2 which required equilibria to be alternative means for achieving the same objective. Of course, whether one, the other or both claims are correct is ultimately an empirical question about linguistic practice. Yet, this does not imply that reasons may not be given in support of adopting one or the other stipulation as part of a more precise and consistent characterization of what the use of the concept of 'convention' is usually intend to convey. That is, of whether to tie the notion of convention to what amounts to the *pursuit of collective ends in social interaction* or to the idea of *alternative interdependent means to one and the same shared end or objective*; or, said differently, whether to tie the notion to the mere multiplicity of equilibria or requiring further that such equilibria should be different ways for achieving one and the same objective.

On the one hand Miller's claim for treating the pursuit of collective ends in social interaction as the basic notion may be supported by the observation that such practices seem to resemble conventions since past conformity is likely to cause expectations of future conformity to these as well. In fact, had this not been the case for the practice emphasized by Miller, the issue could have easily been decided by sticking to Lewis' theory of convention and its reliance on salience by precedent. On the other hand, it is obvious that Miller's conventions-without-alternative are nothing more than artificial or social pratices – i.e. social constructions – falling short from being *arbitrary* in any relevant sense emphasized by the ordinary usage of 'convention'. Thus, Miller's extension of the concept of convention is rejected, while at the same time acknowledging the need to incorporate the presence of a collective end into the definition of a coordination problem:

> **Definition 282.1** *Coordination problem*: a situation of interdependent choice by two or more agents in which there are two or more strict equilibria *where each agent has to base his choice of action upon the expectations about the choices of others as part of a shared attempt of attaining one and the same objective.*

But may this idea of a coordination problem be cashed out exhaustively within a game theoretic framework? Obviously, as was just seen, the requirement of multiple equilibria was not sufficient. Nor, as the *stag hunt* shows, will requiring that these equilibria are strict. But what about returning to the notion of coordination equilibrium? This option may seem particularly attractive since the presence of multiple *coordination* equilibria *does* actually seem to guarantee the presence of an informal coordination problem. That is, the presence of multiple coordination equilibria appears to be sufficient for a coordination problem. Yet, as the *hawk-dove* game revealed, the presence of any such equilibria is not a *necessary* condition which obviously makes that option less attractive.

What ultimately seems to be the problem is that the game theoretic framework only captures the *strategic aspects* of a situation. In particular, it only concerns the agentive preferences over outcomes interpreted as objective states of affairs and not whether these states are states for achieving one and the same objective such as *re-connecting a cut off phone-call*, *avoiding a collision*, or *communicating a particular proposition* unless as this is part of what the agents cares about and consequently becomes part of what determines the preference-ordering. Said differently, game theoretical agents are only concerned with maximizing payoffs and nothing else than such maximization is captured in the models devised for studying strategic interaction. Whether the process of maximization involves one and the same objective, or different ones, are not part of what is sought captured by these models.

Consequently, what needs to be required for characterizing a coordination problem will have to go beyond the game theoretic framework. In particular, in order to characterize such problems it has to be required as a modeling-constraint place upon the attribution of strategy sets that a coordination problem presents multiple equilibria for achieving the same objective. This in turn requires that the strategies attributed to the strategy-sets of agents should be considered as *intentional* ones. The option of going left in the driving game is thus not merely describable as *left*, but as *going-left-in-order-to-avoid-collision*. Obviously, the theorist will often have insider-knowledge as to how to adequately describe these sets and he may make this clear in his description of the situation studied why he usually will leave out these intentions. Still, as well as making the description of a particular coordination problem dependent on psychology, it raises the question of how the theorists as well as the agents arrive at the relevant descriptions. Also, it puts constraints on what strategy-sets may reasonably be attributed to agents as well as what re-descriptions of games are valid for the purpose of various kinds of analysis.